Indian Statistical Institute
B.Math I Year
First Semester , 2005-2006
Mid Semester Examination
Probability Theory I

Time: 2 hrs | Date:13-09-05 | Max. Marks : 70

Note: The paper carries 72 marks. Any score above 70 will be treated as 70.

1. An insurance company insured 2000 scooter drivers, 4000 car drivers, and 6000 truck drivers. The probability of an accident involving a scooter driver, car driver, and a truck driver are respectively 0.01, 0.03, 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver? [12]

2. Suppose n distinguishable balls are put at random into n distinguishable boxes. Show that the probability that exactly one box is empty is $\binom{n}{2} n!/n^n$. [12]

3. 5 cards are drawn at random without replacement from a standard pack of 52 cards. Let X denote the number of aces chosen. find the discrete density function of X. [12]

4. Let X be a real valued discrete random variable; let F denote its distribution function. Let $x \in I\!R$, and $\{y_n\}$ be a sequence such that $y_n \leq y_{n+1} < x$ for all $n = 1, 2, \ldots$ and $\lim_{n\to\infty} y_n = x$. Show that $\bigcup_{n=1}^{\infty} \{X \leq y_n\} = \{X < x\}$, and hence that $F(x) = F(x-) + P(X = x)$. (Here $F(x-) = \lim_{n\to\infty} F(y_n)$.) [16]

5. X, Y are discrete random variables taking values in $\{0, 1, 2, 3, \ldots\}$. Let $Z = \min\{X, Y\}$.

   a) Show that Z is a discrete random variable taking values in $\{0, 1, 2, \ldots\}$.

   b) If X and Y are independent each having a geometric distribution with parameter p, find the distribution of Z. [10 +10]